IA*: An Adjacency-Based Representation for Non-Manifold Simplicial Shapes in Arbitrary Dimensions

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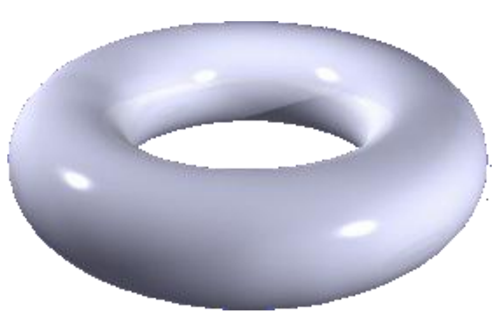
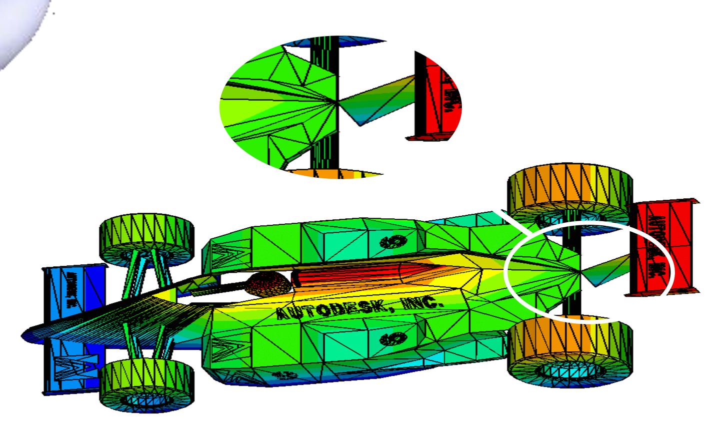
Background

- Need to represent and manipulate 2D, 3D and higher dimensional simplicial complexes describing ***multi-dimensional shapes with complex topology***
- Generalized digital shapes:
  - are discretized through ***simplicial complexes*** over an arbitrary underlying domain
  - can contain ***non-manifold*** singularities
  - can contain ***non-regular*** parts of different dimensionalities



Manifold shape



Non-manifold shape with parts of different dimensionalities

Storage costs

- We compared storage costs of IA* with
  - Incidence-based data structures (***IG*** and ***IS***)
  - Dimension-specific adjacency-based data structures (***TS*** in 2D and ***NMIA*** in 3D)

Model	IG	IS	TS	NMIA	IA*
Armchair	127K	101K	69.4K	-	69.2K
Balance	96K	76K	51.9K	-	51.9K
Carter	95K	75K	53K	-	52K
Chandelier	220K	174K	121K	-	120K
Robot	80K	63K	46K	-	44.9K
Ballon	44K	33K	-	18K	18K
Flasks	104K	74K	-	32	31.8K
Gargoyle	271K	193K	-	83K	83K
Rings	231K	164K	-	68K	67.6K
Teapot	219K	162K	-	84.7K	84K

Storage costs are expressed in terms of the number of pointers

- Over a testbed of 62 manifold, non-regular and non-manifold shapes in 2D and 3D, IA* is the ***most compact*** data structures:
  - 1.5 times ***smaller*** than the IS for 2D models
  - 1.8 times ***smaller*** than the IG for 2D models
  - 2.2 times ***smaller*** than the IS for 3D models
  - 3.2 times ***smaller*** than the IG for 3D models
  - 5% ***smaller*** than the TS for 2D models
  - 3% ***smaller*** than the NMIA for 3D models

Contribution

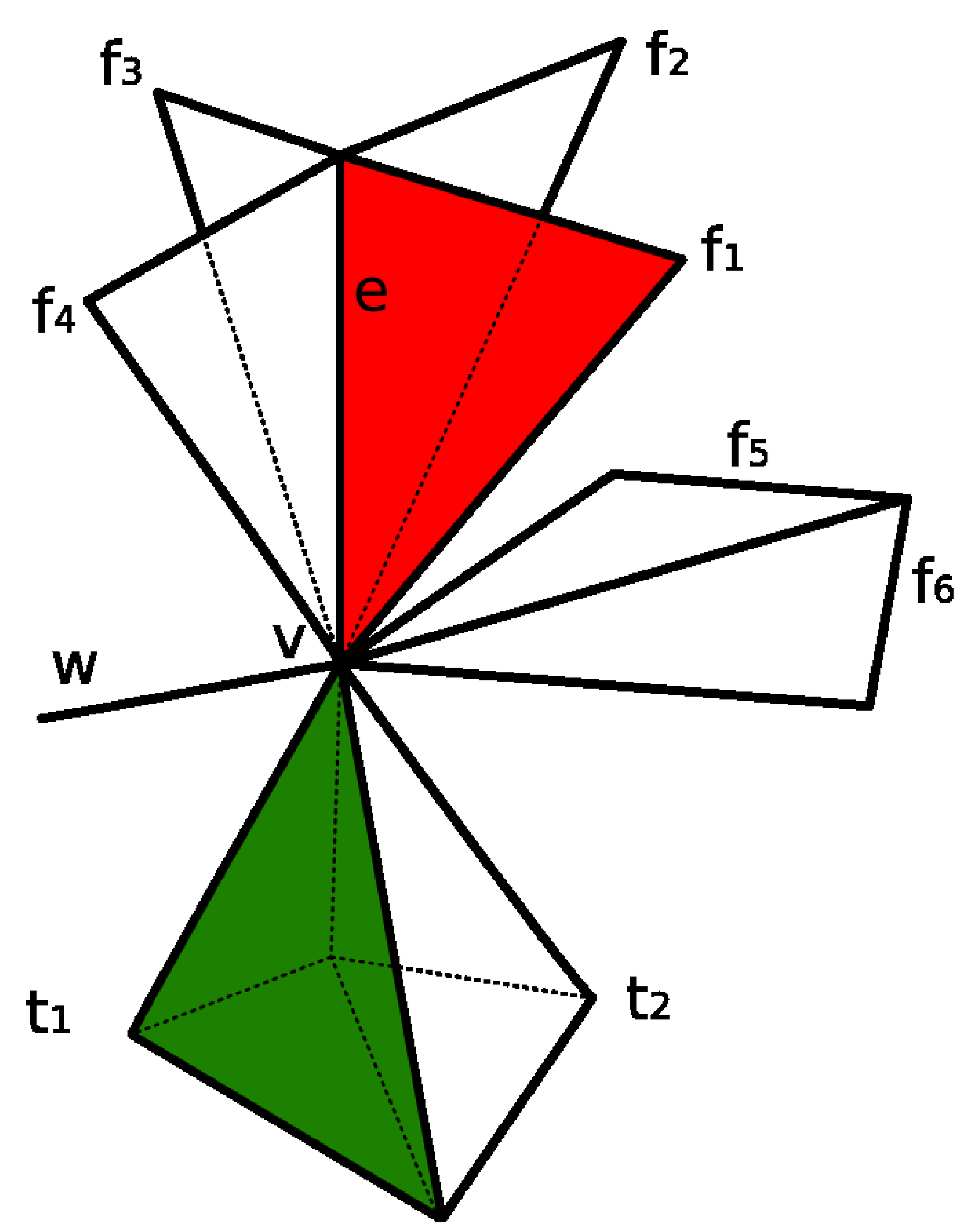
- The ***Generalized Indexed data structure with Adjacencies (IA*):***
  - dimension-independent adjacency-based data structure for general shapes
  - agnostic about ***embedding*** of the input shape in the underlying space
  - encodes only ***vertices*** and ***top simplices*** (simplices not on the boundary of other simplices)
  - ***optimal retrieval*** of all topological relations
  - ***scalable*** with respect to manifold case
  - supports ***shape editing*** operations
  - ***more compact*** than state of the art
    - dimension-independent incidence-based ***Incidence Graph (IG)*** *[Ede87] and* ***Incidence Simplicial (IS)*** *[DFHPC10]*
    - dimension-specific adjacency-based ***Triangle Segment (TS)*** *[DFMPS04]* in 2D ***Non-manifold Incidence with Adjacency (NMIA)*** *[DFMPS04] in 3D*

Topological queries

- ***Boundary relations*** for p-simplex σ are retrieved by generating faces of σ, requiring constant time:
  - IA* is 15% ***faster*** than IG and IS
- ***Co-boundary relations*** of type $\mathbf{R_{0,q}(v)}$ are retrieved with respect to top simplices incident in v, requiring time linear in the number of *top simplices* in the *star* of vertex *v:*
  - IA* is 20% ***faster*** than IG and 30% ***faster*** than IS for 2D models
  - IA* is 35% ***faster*** than IG and 62% ***faster*** than IS for 3D models
- ***Co-boundary relations*** of type $\mathbf{R_{p,q}(\sigma)}$, with $p \neq 0$, are based on the retrieval of the $\mathbf{R_{0,q}(v)}$ relation for a vertex v of simplex σ, requiring time linear in the number of *top simplexes* incident in v:
  - IA* is 15% *slower* than IG for $R_{1,q}$
  - IA* is 11% *slower* than IS for $R_{1,q}$
- ***Adjacency relations*** for a simplex σ are retrieved by combining boundary and co-boundary relations and require time linear in the number of top simplices incident in one vertex of σ.

Representation

- Encode only ***vertices*** and ***top simplices***
- For each vertex v:
  - $\mathbf{R^*_{0,1}(v)}$: all top 1-simplices incident in v
  - $\mathbf{R^*_{0,p}(v)}$: one top p-simplex (with $p>1$) for each (p-1)-connected component of the *link* of v
- For each top p-simplex σ:
  - $\mathbf{R_{p,0}(\sigma)}$: all vertices in the boundary of σ
  - $\mathbf{R^*_{p,p}(\sigma)}$: all top p-simplices adjacent to σ along a $(p-1)$-face of σ
- Key observation
  - Encode top p-simplices in non-manifold singularities along $(p-1)$-faces of σ *collectively* through $\mathbf{R^*_{p-1,p}(\sigma)}$ relation
- IA* reverts to ***IA data structure*** *[PBCF93]* when presented with manifold shape



$$R^*_{0,1}(v) = \{w\},$$
$$R^*_{0,2}(v) = \{f_1, f_5\},$$
$$R^*_{0,3}(v) = \{t_1\}$$
$$R^*_{2,2}(f_1) = R^*_{1,2}(e) = \{f_1, f_2, f_3, f_4\}$$

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