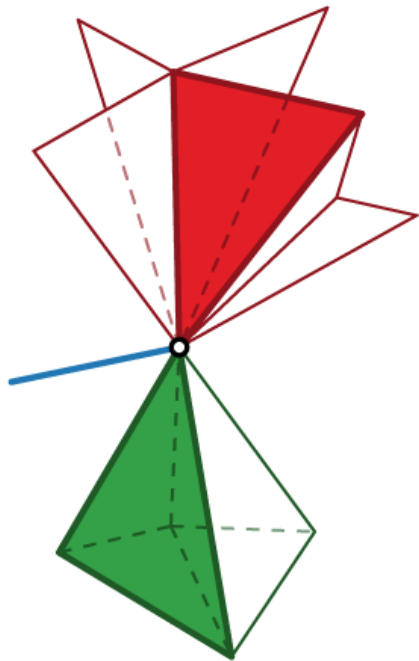


IA*:

AN ADJACENCY-BASED REPRESENTATION FOR NON-MANIFOLD SIMPLICIAL SHAPES IN ARBITRARY DIMENSIONS



David Canino

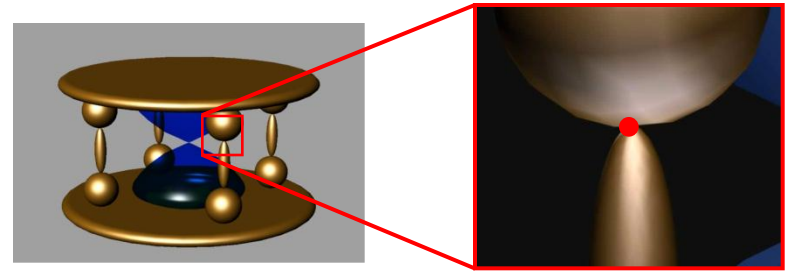
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University of Genova

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MOTIVATION



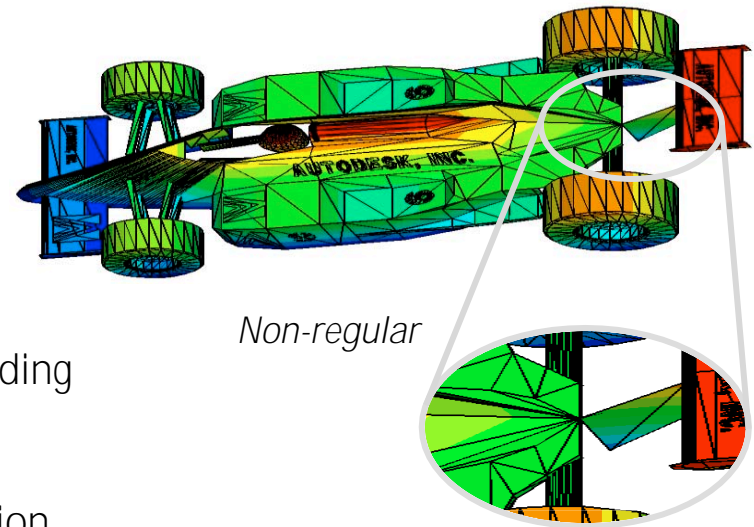
Non-manifold singularity

∅ Generalized digital shapes:

- ∅ are discretized through *simplicial complexes* over an arbitrary underlying domain
- ∅ can contain *non-manifold* singularities
- ∅ can contain *non-regular* parts of different dimensionalities

∅ Arise in many processes

- ∅ Intentional
 - ∅ e.g. idealization process, shape understanding
- ∅ Unintentional
 - ∅ e.g. during mesh generation or manipulation



Non-regular

DATA STRUCTURES FOR SIMPLICIAL MESHES

Taxonomy (*partial*)

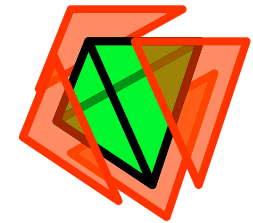
- *Dimension-specific vs. dimension-independent*
- *Manifold vs. non-manifold vs. non-regular*
- *Incidence-based vs. adjacency-based*
- *Efficient support for topological relations*

TOPOLOGICAL RELATIONS

∅ Describe the *connectivity*

&_ _ Boundary relations (\rightarrow \rightarrow)

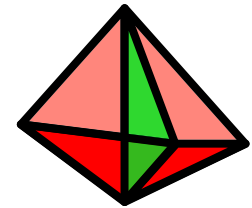
∅ Set of \rightarrow -simplices that are a face of a given \rightarrow -simplex



&

&_ _ Co-boundary relations (\rightarrow \rightarrow)

∅ Set of simplices that have a given simplex as a face

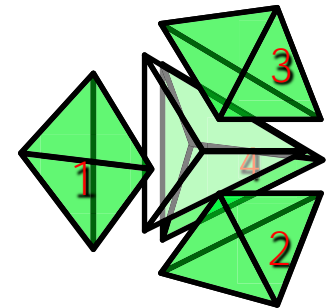


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&_ _ Adjacency relations

∅ Set of \rightarrow -simplices that adjacent to a given simplex along a \rightarrow face (\rightarrow \rightarrow)

∅ Set of vertices connected by an edge (\rightarrow \rightarrow)



&

IA*: GENERALIZED INDEXED DATA STRUCTURE WITH ADJACENCIES

- ∅ *Adjacency-based* representation
- ∅ *General shapes*
 - ∅ Allows manifold, non-regular and non-manifold
- ∅ *Dimension-independent*
 - ∅ \ddot{Y} -dimensional shapes in \mathbb{R}^a , \mathbb{Y}^a
 - ∅ Agnostic about *embedding* in underlying space
- ∅ *Efficient retrieval* of all topological relations
- ∅ *Scalable* with respect to manifold case
 - ∅ No overhead in manifold regions
- ∅ Supports *shape editing* operations
- ∅ *Compact encoding*
 - ∅ with respect to the state of the art

REPRESENTATION

∅ Entities

∅ Vertices

∅ *Top simplices*

∅ Simplices not on boundary of another simplex

∅ Encoded in terms of their vertices

∅ Topological Relations

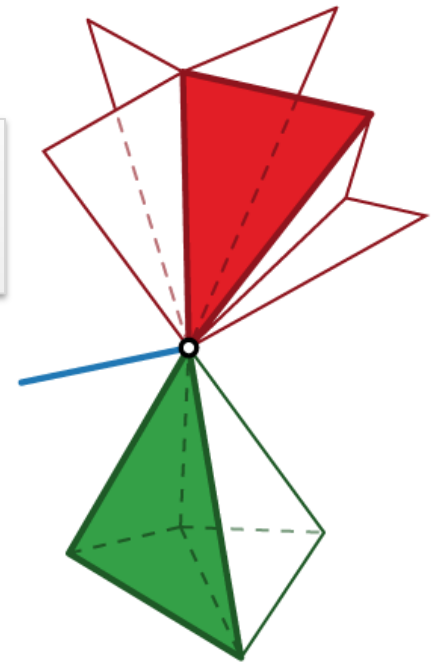
∅ & ξ Boundary relations for *top* ξ -simplices ($\xi < 3$)

∅ & ξ Partial co-boundary relations for vertices ($\xi = 0$)
 One top simplex in each ($\xi = 0$)-connected component in link

∅ & $\xi\xi$ Adjacency relations for *top* ξ -simplices ($\xi < 3$)

∅ & $\xi \xi$ Partial co-boundary relations for non-manifold ξ -simplices incident to top ξ -simplices ($\xi < 3$)

1 top edge
 5 top triangles
 2 top tetrahedra



STORAGE RESULTS (HIGHLIGHTS)

- Compared to state of the art
 - *Dimension-independent, incidence-based* representation
 - IG** – Incidence Graph
 - IS** – Incidence Simplicial
 - *Dimension-specific, adjacency-based* representation
 - TS** – Triangle-Segment ($d=2$ in \mathbb{R}^3)
 - NMIA** – Non-manifold incidence-based data structure with Adjacencies ($d=3$ in \mathbb{R}^3)
- Testbed of 62 datasets
 - $d=\{2,3\}$ in \mathbb{R}^3
 - *manifold, non-manifold and non-regular*

STORAGE RESULTS (HIGHLIGHTS)

$d=2$ in R^3

- ~1.8 times *smaller* than **IG**
- ~1.5 times *smaller* than **IS**
-
- ~5% *smaller* than **TS**

$d=3$ in R^3

- ~3.2 times smaller than **IG**
- ~2.2 times smaller than **IS**
-
- ~3% smaller than **NMIA**

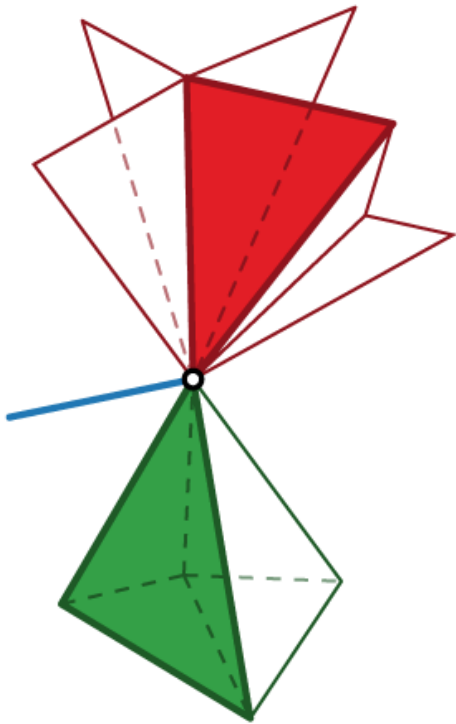
➤ IA* is most compact in all cases

QUERYING RESULTS (HIGHLIGHTS)

- Boundary relations
 - Expressed as *tuples* of vertices in constant time
 - 15% *faster* than state of the art incidence-based representations
- Co-boundary relations
 - $R_{0,k}(v)$ – Retrieved w.r.t top simplices incident to vertex in time linear in star of vertex
 - 20-30% *faster* in 2D; 30-60% faster in 3D
 - $R_{j,k}(\sigma)$ – based on retrieval of a vertex in boundary of σ
 - 10-15% *slower* than incidence-based representations
- Adjacency relations
 - $R_{k,k}(\sigma)$ – combine boundary and co-boundary relations
 - Time is linear in number of simplices in star of a vertex of σ

CONCLUSION

- First *adjacency-based, dimension-independent* approach for *general simplicial meshes*
- Most compact topological representation for general meshes
 - No storage overhead with respect to **IA** data structure when presented with manifold dataset
- Does not encode non-top simplices
 - Might not be applicable in certain applications
 - e.g. finite element analysis
- Supports editing operations (not discussed)
 - Vertex-pair contraction
- Plan to release as part of C++ open source meshing library
 - **Mangrove TDS**



THANK YOU

Anonymous reviewers

National Science Foundation