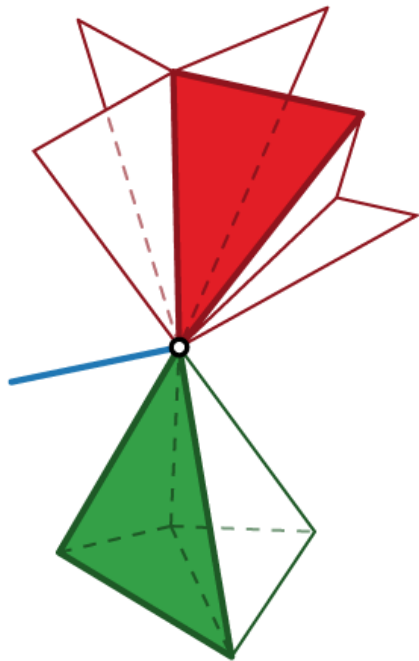


# IA\*:

## AN ADJACENCY-BASED REPRESENTATION FOR NON-MANIFOLD SIMPLICIAL SHAPES IN ARBITRARY DIMENSIONS



David Canino

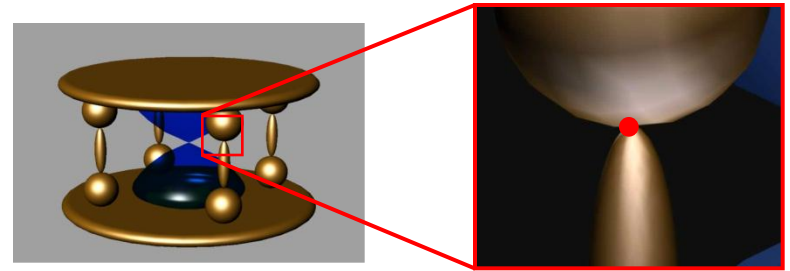
Leila De Floriani

University of Genova

**Kenneth Weiss**

University of Maryland, College Park

# MOTIVATION



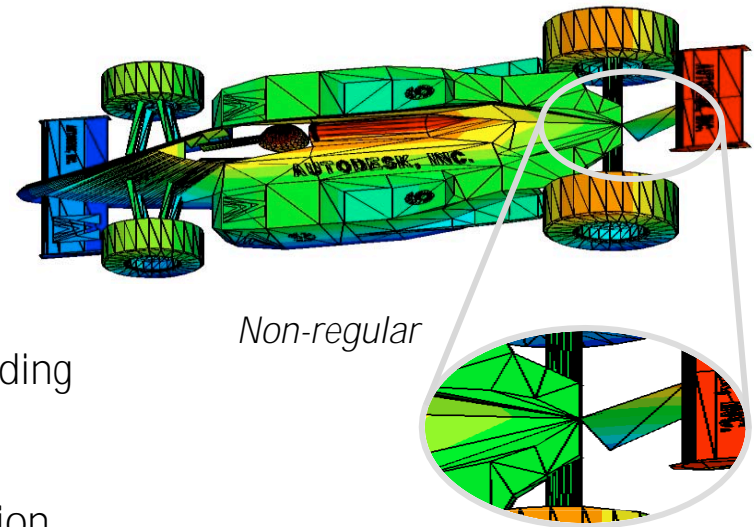
*Non-manifold singularity*

## ∅ Generalized digital shapes:

- ∅ are discretized through *simplicial complexes* over an arbitrary underlying domain
- ∅ can contain *non-manifold* singularities
- ∅ can contain *non-regular* parts of different dimensionalities

## ∅ Arise in many processes

- ∅ Intentional
  - ∅ e.g. idealization process, shape understanding
- ∅ Unintentional
  - ∅ e.g. during mesh generation or manipulation



*Non-regular*

# DATA STRUCTURES FOR SIMPLICIAL MESHES

## Taxonomy (*partial*)

- *Dimension-specific vs. dimension-independent*
- *Manifold vs. non-manifold vs. non-regular*
- *Incidence-based vs. adjacency-based*
- *Efficient support for topological relations*

# TOPOLOGICAL RELATIONS

∅ Describe the *connectivity*

&\_ \_ Boundary relations ( $\rightarrow$   $\rightarrow$ )

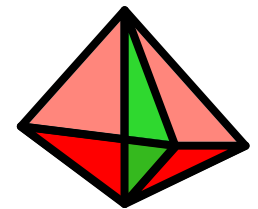
∅ Set of  $\rightarrow$ -simplices that are a face of a given  $\rightarrow$ -simplex



&

&\_ \_ Co-boundary relations ( $\rightarrow$   $\rightarrow$ )

∅ Set of simplices that have a given simplex as a face

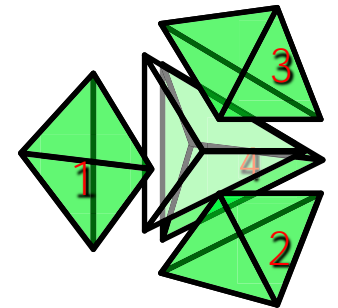


&

&\_ \_ Adjacency relations

∅ Set of  $\rightarrow$ -simplices that adjacent to a given simplex along a  $\rightarrow$  face ( $\rightarrow$   $\rightarrow$ )

∅ Set of vertices connected by an edge ( $\rightarrow$   $\rightarrow$ )



&

# IA\*: GENERALIZED INDEXED DATA STRUCTURE WITH ADJACENCIES

- ∅ *Adjacency-based* representation
- ∅ *General shapes*
  - ∅ Allows manifold, non-regular and non-manifold
- ∅ *Dimension-independent*
  - ∅  $\ddot{Y}$ -dimensional shapes in  $\mathbb{R}^a$ ,  $\mathbb{Y}^a$
  - ∅ Agnostic about *embedding* in underlying space
- ∅ *Efficient retrieval* of all topological relations
- ∅ *Scalable* with respect to manifold case
  - ∅ No overhead in manifold regions
- ∅ Supports *shape editing* operations
- ∅ *Compact encoding*
  - ∅ with respect to the state of the art

# REPRESENTATION

## ∅ Entities

∅ Vertices

∅ *Top simplices*

∅ Simplices not on boundary of another simplex

∅ Encoded in terms of their vertices

## ∅ Topological Relations

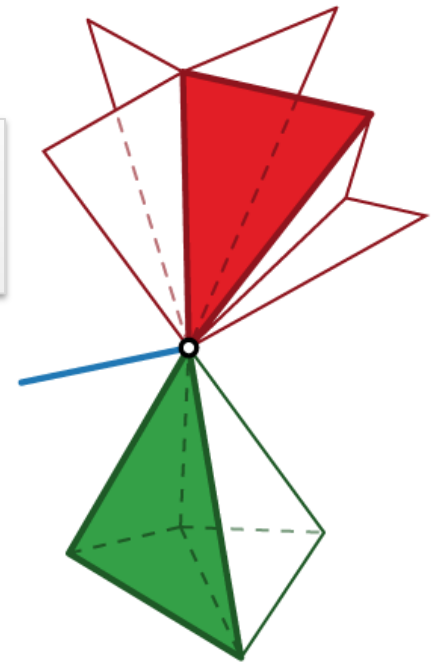
∅  $\& \xi$  Boundary relations for *top*  $\xi$ -simplices ( $\xi \geq 1$ )

∅  $\& \xi$  Partial co-boundary relations for vertices ( $\xi = 0$ )  
 One top simplex in each ( $\xi = 0$ )-connected component in link

∅  $\& \xi \xi$  Adjacency relations for *top*  $\xi$ -simplices ( $\xi \geq 1$ )

∅  $\& \xi \xi$  Partial co-boundary relations for non-manifold  $\xi$ -simplices incident to top  $\xi$ -simplices ( $\xi \geq 1$ )

1 top edge  
 5 top triangles  
 2 top tetrahedra





# STORAGE RESULTS (HIGHLIGHTS)

- Compared to state of the art
  - *Dimension-independent, incidence-based* representation
    - IG** – Incidence Graph
    - IS** – Incidence Simplicial
  - *Dimension-specific, adjacency-based* representation
    - TS** – Triangle-Segment ( $d=2$  in  $\mathbb{R}^3$ )
    - NMIA** – Non-manifold incidence-based data structure with Adjacencies ( $d=3$  in  $\mathbb{R}^3$ )
- Testbed of 62 datasets
  - $d=\{2,3\}$  in  $\mathbb{R}^3$
  - *manifold, non-manifold and non-regular*



# STORAGE RESULTS (HIGHLIGHTS)

$d=2$  in  $R^3$

- ~1.8 times *smaller* than **IG**
- ~1.5 times *smaller* than **IS**
- 
- ~5% *smaller* than **TS**

$d=3$  in  $R^3$

- ~3.2 times smaller than **IG**
- ~2.2 times smaller than **IS**
- 
- ~3% smaller than **NMIA**

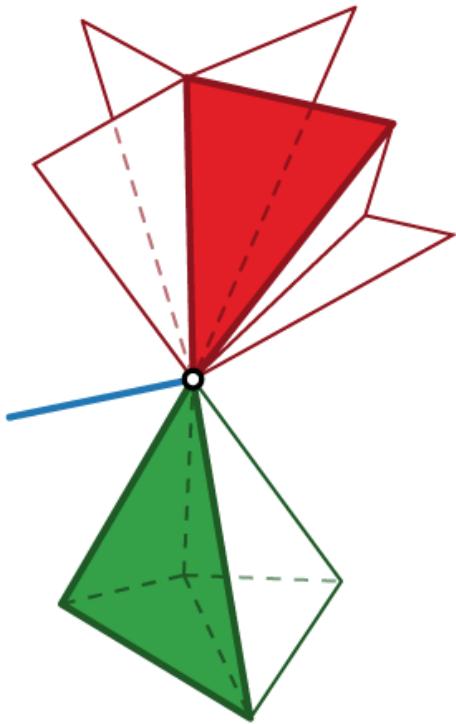
➤ IA\* is most compact in all cases

# QUERYING RESULTS (HIGHLIGHTS)

- Boundary relations
  - Expressed as *tuples* of vertices in constant time
  - 15% *faster* than state of the art incidence-based representations
- Co-boundary relations
  - $R_{0,k}(v)$  – Retrieved w.r.t top simplices incident to vertex in time linear in star of vertex
    - 20-30% *faster* in 2D; 30-60% faster in 3D
  - $R_{j,k}(\sigma)$  – based on retrieval of a vertex in boundary of  $\sigma$ 
    - 10-15% *slower* than incidence-based representations
- Adjacency relations
  - $R_{k,k}(\sigma)$  – combine boundary and co-boundary relations
  - Time is linear in number of simplices in star of a vertex of  $\sigma$

# CONCLUSION

- First *adjacency-based, dimension-independent* approach for *general simplicial meshes*
- Most compact topological representation for general meshes
  - No storage overhead with respect to **IA** data structure when presented with manifold dataset
- Does not encode non-top simplices
  - Might not be applicable in certain applications
    - e.g. finite element analysis
- Supports editing operations (not discussed)
  - Vertex-pair contraction
- Plan to release as part of C++ open source meshing library
  - **Mangrove TDS**



# THANK YOU

Anonymous reviewers

National Science Foundation