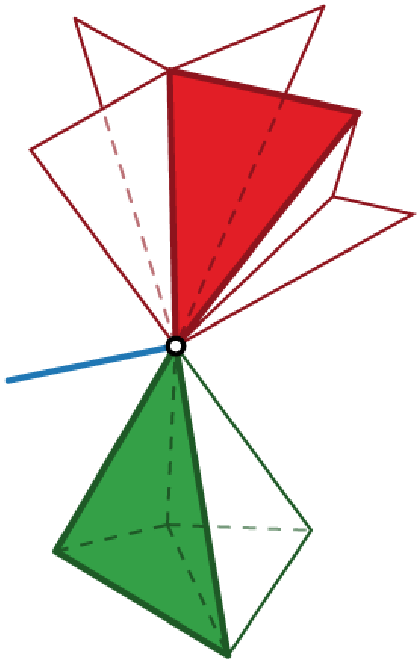
IA*: An Adjacency-Based Representation for Non-Manifold Simplicial Shapes in Arbitrary Dimensions



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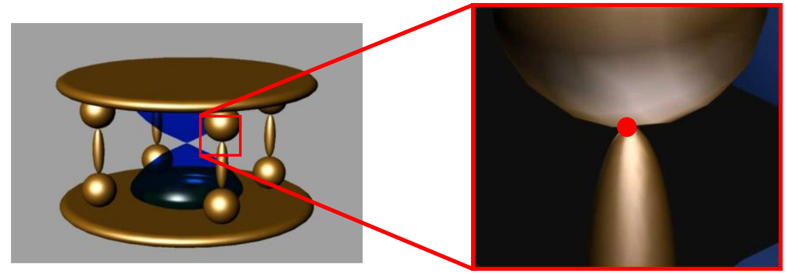
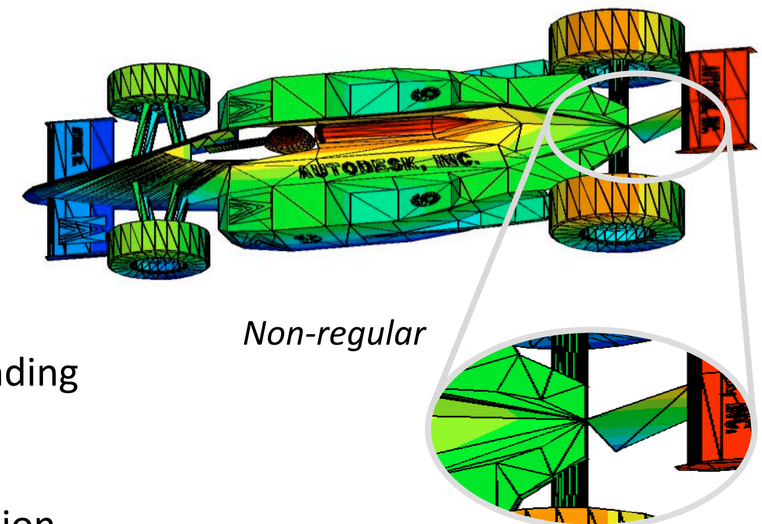
MOTIVATION



Non-manifold singularity

- Generalized digital shapes:
  - are discretized through ***simplicial complexes*** over an arbitrary underlying domain
  - can contain ***non-manifold*** singularities
  - can contain ***non-regular*** parts of different dimensionalities
- Arise in many processes
  - Intentional
    - e.g. idealization process, shape understanding
  - Unintentional
    - e.g. during mesh generation or manipulation



Non-regular

DATA STRUCTURES FOR SIMPLICIAL MESHES

Taxonomy *(partial)*

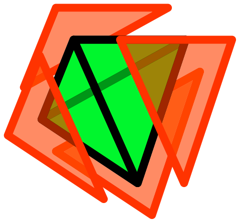
- *Dimension-specific* vs. *dimension-independent*
- *Manifold* vs. *non-manifold* vs. *non-regular*
- *Incidence-based* vs. *adjacency-based*
- Efficient support for *topological relations*

Topological Relations

- Describe the *connectivity* of the mesh's elements

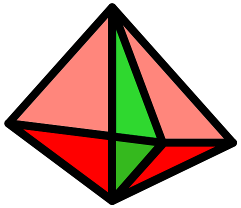
$R_{p,q}$ – Boundary relations ($p<q$)

- Set of q-simplices that are a face of a given p-simplex



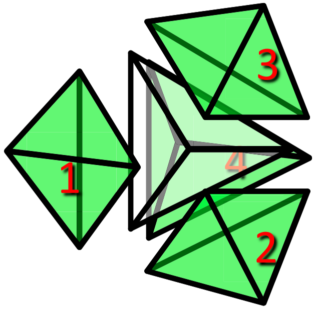
$R_{q,p}$ – Co-boundary relations ($p<q$)

- Set of simplices that have a given simplex as a face



$R_{p,p}$ – Adjacency relations

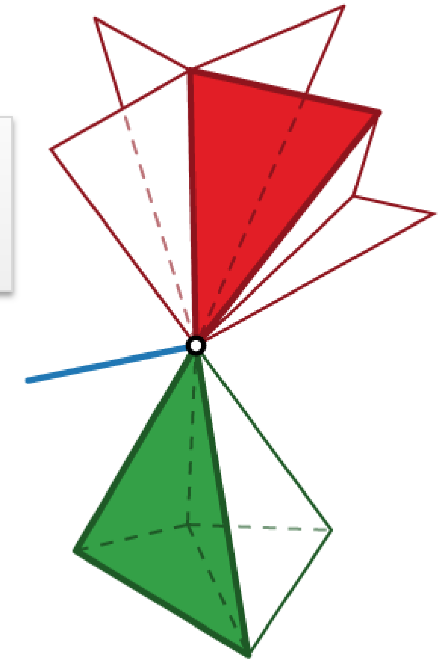
- Set of p-simplices that adjacent to a given simplex along a p-1 face ($p>0$)
- Set of vertices connected by an edge ($p=0$)



IA*: Generalized Indexed Data Structure with Adjacencies

- *Adjacency-based* representation
- *General shapes*
  - Allows manifold, non-regular and non-manifold
- *Dimension-independent*
  - d-dimensional shapes in R^n, $d \leq n$
  - Agnostic about ***embedding*** in underlying space
- ***Efficient retrieval*** of all topological relations
- ***Scalable*** with respect to manifold case
  - No overhead in manifold regions
- Supports ***shape editing*** operations
- ***Compact encoding***
  - with respect to the state of the art

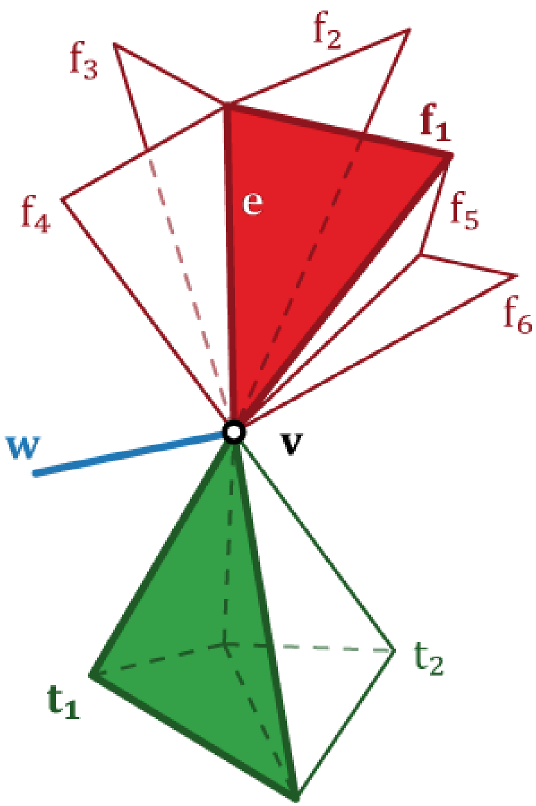
REPRESENTATION



- Entities
  - Vertices
  - ***Top simplices***
    - Simplices not on boundary of another simplex
    - Encoded in terms of their vertices

- Topological Relations
  - $R^*_{k,0}$ – Boundary relations for ***top*** k-simplices ($k>0$)
  - $R^*_{0,k}$ – Partial co-boundary relations for vertices ($k>0$)
    One top simplex in each *(k-1)*-connected component in link
  - $R^*_{k,k}$ – Adjacency relations for ***top*** k-simplices ($k>1$)
  - $R^*_{k-1,k}$ – Partial co-boundary relations for non-manifold
    k-1 simplices incident to top k-simplices ($k>1$)

EXAMPLE



$$R^*_{0,1}(v) = \{\, w \,\}$$

$$R^*_{0,2}(v) = \{\, f_1 \,\}$$

$$R^*_{0,3}(v) = \{\, t_1 \,\}$$

$$R^*_{2,2}(\, f_1 \,) = \{R^*_{1,2}(\, e \,), f_5 , \emptyset\}$$

$$R^*_{1,2}(\, e \,) = \{\, f_1 , f_2 , f_3 , f_4 \}$$

Key observation: Encode collection of top *p*-simplices incident to a non-manifold *p-1* simplex as a single unit

Storage Results (Highlights)

- Compared to state of the art
  - *Dimension-independent, incidence-based* representation

    IG – Incidence Graph

    IS – Incidence Simplicial
  - *Dimension-specific, adjacency-based* representation

    TS –Triangle-Segment ($d=2$ in R^3)

    NMIA –Non-manifold incidence-based data structure with Adjacencies ($d=3$ in R^3)
- Testbed of 62 datasets
  - d={2,3} in R^3
  - *manifold*, *non-manifold* and *non-regular*

Storage Results (Highlights)

d=2 in R^3

- ~1.8 times *smaller* than ***IG***
- ~1.5 times *smaller* than ***IS***
  -
- ~5% *smaller* than ***TS***

d=3 in R^3

- ~3.2 times smaller than ***IG***
- ~2.2 times smaller than ***IS***
  -
- ~3% smaller than ***NMIA***

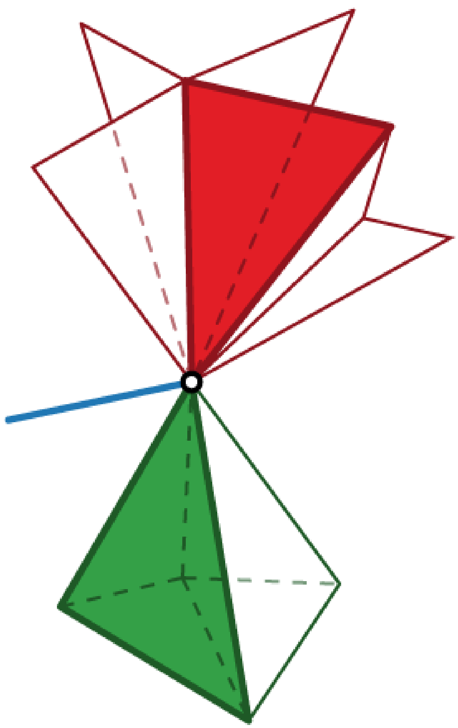
➢ IA* is most compact in all cases

Querying Results (Highlights)

- Boundary relations
  - Expressed as *tuples* of vertices in constant time
  - 15% *faster* than state of the art incidence-based representations
- Co-boundary relations
  - $R_{0,k}(v)$ – Retrieved w.r.t top simplices incident to vertex in time linear in star of vertex
    - 20-30% ***faster*** in 2D; 30-60% faster in 3D
  - $R_{j,k}(\sigma)$ – based on retrieval of a vertex in boundary of σ
    - 10-15% *slower* than incidence-based representations
- Adjacency relations
  - $R_{k,k}(\sigma)$ – combine boundary and co-boundary relations
  - Time is linear in number of simplices in star of a vertex of σ

CONCLUSION

- First *adjacency-based, dimension-independent* approach for *general simplicial meshes*
- Most compact topological representation for general meshes
  - No storage overhead with respect to ***IA*** data structure when presented with manifold dataset
- Does not encode non-top simplices
  - Might not be applicable in certain applications
    - e.g. finite element analysis
- Supports editing operations (not discussed)
  - Vertex-pair contraction
- Plan to release as part of C++ open source meshing library
  - ***Mangrove TDS***



Thank You

Anonymous reviewers

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